[redacted]

Write name and student number on each page!

Exam
SOLID MECHANICS (NASM)
January 26, 2016, 09:00–12:00 h

This exam comprises four problems, for which one can obtain the following points:

Question	# points
1	1+0.5=1.5
2	2+1=3
3	2+2+1+1=6
4	1+2=3

The final grade is calculated as $(\#\ \text{points} + 1.5)/1.5$.

Question 1 In 1913, Von Mises proposed the following scalar representation of a stress tensor:

$$\sigma_{\mathrm{v}} = \sqrt{\frac{3}{2}\boldsymbol{\sigma}' : \boldsymbol{\sigma}'},$$

where, as usual,

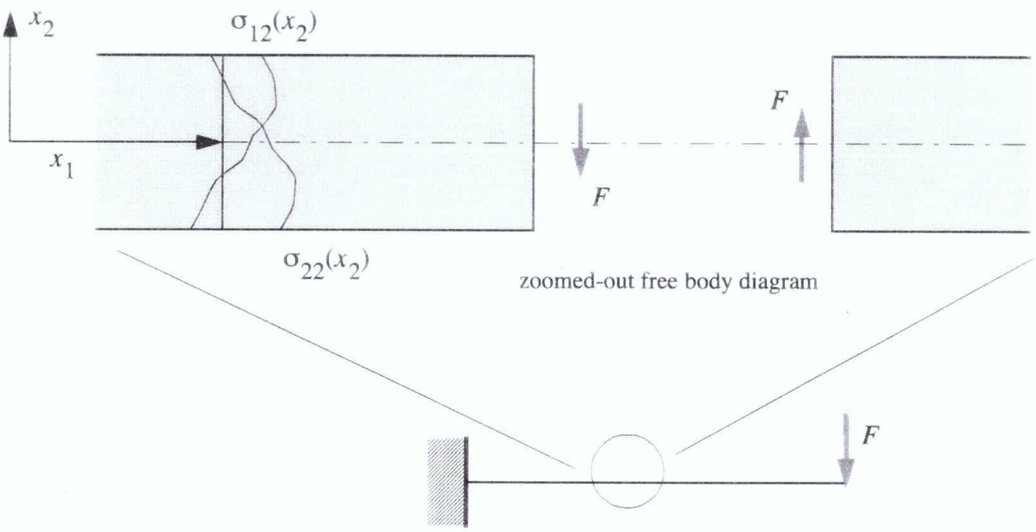
$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \left(\frac{1}{3}\operatorname{tr}\boldsymbol{\sigma}\right)\boldsymbol{I}$$

is the stress deviator.

a. Compute σ_{v} for uniaxial tension at a stress σ, as well as for pure shear $\sigma_{12} = \sigma_{21} = \tau$, otherwise $\sigma_{ij} = 0$.

b. According to Mohr's circle discussed in Chapter 2.6, pure shear can be regarded as a combination of tension and compression in two mutually orthogonal directions. Does this also hold for the corresponding Von Mises stresses from (a) ~~en (b)~~? Explain.

Question 2 When a cantilever is loaded by a force F at its end, as in the second forget-me-not in Fig. 3.6, there is a moment in the beam that leads to bending. However, at any cross-section of the beam there is also a force which, because of force equilibrium, is always equal to F, see figure below. While the bending moment (not drawn here for clarity, but seen in Fig. 3.5 of the



textbook) leads to the linear distribution of σ_{11} over x_2 as given in Eq. (3.45), the lateral force will have to be equilibrated by a shear stress distribution $\sigma_{12}(x_2)$ along any cross section. This has not discussed been discussed in Sec. 3.7.2 but is present. Also it cannot be ruled out that there is a distribution of σ_{22}. Let us investigate to what extent this is taken into account in the so-called Euler-Bernoulli beam theory that we have been using.

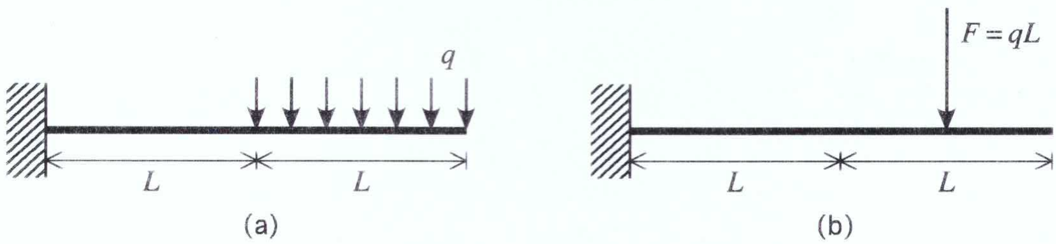
a. Determine the shear stress field that is implied by the solution of the second forget-me-not,

$$w(x) = \frac{F}{EI}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right).$$

Discuss your answer in relation to the boundary conditions along the top and bottom of the beam, and the net vertical force transmitted across cross sections. (NB1: the deflection w gives the vertical displacement of the neutral axis in the $-x_2$ direction, cf. page 52 of the book. NB2: identify x in the solution for w as x_1 in the detailed view).

b. What can you say about σ_{22}?

Question 3 Although it is convenient to solve beam bending problems by assuming that loads are concentrated forces, in reality load is always distributed over some area. To get a very crude idea of the error made due to this assumption, we consider a cantilever of length $2L$ with bending stiffness EI under two kinds of loading: (a) a uniformly distributed load q over the last half of its length, (b) the same resulting load qL as an equivalent force.



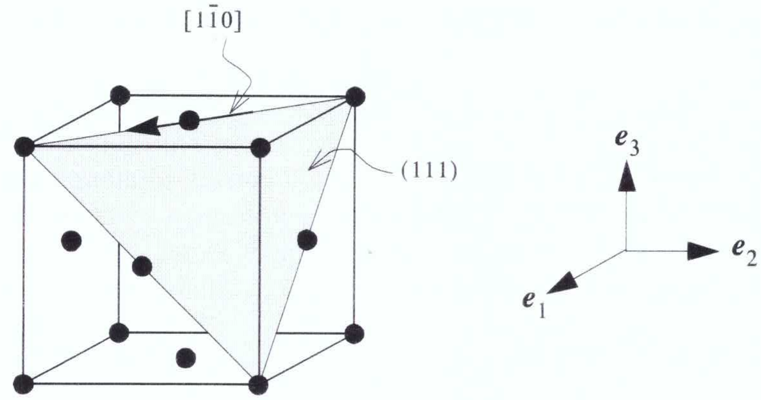
a. Determine the distribution of the bending moment for both cases and discuss the salient differences.

b. Compute the end deflection for case (a) by using superposition and the third forget-me-not.

c. Express the end deflection for case (b) in terms of q, L and EI. Compare your answer with that to question (b).

d. Compare the deflections for case (a) and (b) halfway the beam, i.e. at a distance L from the wall.

Question 4 An fcc crystal is known to have 12 slip systems: 3 different slip directions on each of 4 equivalent slip planes. One example is indicated in the figure below by means of Miller indices.



In terms of components with respect to the orthonormal basis $\{\boldsymbol{e}_i\}$ $(i = 1, 2, 3)$ this $(111)[1\bar{1}0]$ slip system is characterized by the slip plane normal vector $\boldsymbol{m}$ and slip direction $\boldsymbol{s}$ given as

$$m = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad s = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

The following table lists all slip systems in terms of Miller indices and their usual names in the materials science community:

name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	$(1\bar{1}1)$	$(1\bar{1}1)$	$(1\bar{1}1)$
direction	$[\bar{1}01]$	$[0\bar{1}1]$	$[\bar{1}10]$	$[\bar{1}01]$	$[011]$	$[110]$
name	A2	A6	A3	C5	C3	C1
plane	$(\bar{1}11)$	$(\bar{1}11)$	$(\bar{1}11)$	$(11\bar{1})$	$(11\bar{1})$	$(11\bar{1})$
direction	$[0\bar{1}1]$	$[110]$	$[101]$	$[\bar{1}10]$	$[101]$	$[011]$

When this crystal is subjected to uniaxial tension in the $[001]$ direction, i.e., $\boldsymbol{\sigma} = \sigma \boldsymbol{e}_3 \otimes \boldsymbol{e}_3$, 8 out of 12 slip systems will be activated (this does not need to be shown).

Here we will consider uniaxial tension in the $[011]$ direction, i.e. parallel to $\boldsymbol{e}_2 + \boldsymbol{e}_3$.

a. Show that this state is represented by the stress components

$$[\sigma_{ij}] = \frac{1}{2}\sigma \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

with respect to $\{\boldsymbol{e}_i\}$.

b. How many slip systems are active when the crystal is subjected to uniaxial tension in the $[011]$ direction?